SV-FBP for Non-Circular CBCT via Trajectory Standardization: Learn Once and for All

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***Abstract*—Differentiable shift-variant filtered backprojection (SV-FBP) is a fast and efficient reconstruction algorithm for cone-beam computed tomography with non-circular trajectories. While this method is significantly faster than iterative methods, its reliance on trajectory-specific training of Radon-domain weights limits its ability to generalise and reuse it across different scanning geometries. In this paper, we propose a novel training framework that facilitates trajectory-generalizable learning for SV-FBP. The fundamental concept underpinning this methodology is a preprocessing pipeline that standardises a set of CBCT trajectories by reordering projections based on geometric continuity. This standardization decouples the learned SV-FBP weights from the absolute trajectory configuration, thereby making them invariant to projection order, allowing a single model to be reused across multiple trajectories. Experimental results show that the proposed method enables a single SV-FBP model to generalize across four distinct non-circular CBCT trajectories, achieving consistent performance with an average MSE of 0.1207, PSNR of 34.68 dB, and SSIM of 0.9177, without the need for retraining. This confirms the effectiveness of the standardization framework in enhancing flexibility and scalability.**

***Index Terms*—CBCT Reconstruction, Deep Learning, Known Operator, Arbitrary Trajectory, Geometry Standardization.**

I. Introduction

Cone-beam computed tomography (CBCT) has become an important imaging modality in interventional radiology, image-guided surgery, and radiation therapy, owing to its ability to acquire three-dimensional (3D) volumetric data using a single rotation of an X-ray source and a flat-panel detector. Conventional CBCT systems typically follow a circular source trajectory, enabling analytical reconstruction algorithms such as the Feldkamp-Davis-Kress (FDK) method [1] to generate high-quality images under the assumption of approximate data completeness.

However, in many practical scenarios, particularly those requiring flexible imaging geometries, such as robotic C-arm systems, non-circular trajectories are increasingly adopted. These trajectories are often task-driven, tailored to enhance imaging performance for specific clinical objectives. For example, sinusoidal trajectories can reduce metal artefacts; reverse helical paths can extend the longitudinal field of view (FOV); and task-optimized trajectories can increase the detectability of anatomical structures or lesions [2]. While this flexibility improves diagnostic utility, it also introduces considerable challenges for accurate and efficient image reconstruction.

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Iterative reconstruction algorithms, such as algebraic reconstruction techniques (ART) and model-based iterative reconstruction (MBIR), are capable of handling these irregular geometries by explicitly modeling the system matrix. However, their high computational cost and memory demand significantly limit their applicability in time-sensitive or resource-constrained clinical scenarios.

To address these limitations, the shift-variant filtered backprojection (SV-FBP) algorithm [3] has emerged as a promising alternative, offering a balance between reconstruction quality and computational efficiency. Building upon this concept, Ye *et al.* [4] further advanced SV-FBP by mapping it as a neural network using known operator learning [5]. In this method, the reconstruction pipeline remains largely analytical, with only the trajectory-dependent weights in the filtering parameterized as learnable. These weights can be efficiently optimized from training data using backpropagation, thereby enabling data-driven adaptation of the reconstruction process to non-circular geometries. However, a key limitation of differentiable SV-FBP lies in its lack of generalizability, as the learned weights are tightly coupled to the specific scan trajectory.

In this paper, we address this limitation by proposing a novel preprocessing and training framework that enables differentiable SV-FBP to generalize across geometry-consistent trajectories. Our approach transforms a set of CBCT trajectories into a unified canonical form by identifying a consistent starting view and enforcing spatial continuity across projections. This standardization resolves inconsistencies in projection ordering and decouples the learned reconstruction weights from absolute trajectory configuration. This decoupling allows a single trained SV-FBP model to generalize across a wide range of trajectories, significantly improving scalability in settings with variable scan paths.

II. Methods

The proposed training framework generalizes the differentiable SV-FBP model across CBCT trajectories that share a consistent geometric structure. This is achieved by implementing a trajectory standardisation procedure prior to training and inference.

A. Differentiable Shift-Variant FBP model

In prior work [4], we introduced a differentiable formulation of the SV-FBP algorithm tailored for arbitrary trajectoriy CBCT reconstruction:

$$f(x) = A_{3d}^T w_d A_{2d}^T D w_{red} w_{sino} D A_{2d} w_{cos} g(x, y, \lambda). \tag{1}$$

This formulation can be interpreted as a trajectory-dependent shift-variant filtering process followed by 3D cone-beam backprojection A_{3d}^T. The filtering pipeline comprises a series of known operators: cosine weighting w_{cos}, 2D Radon transform A_{2d}, differentiation D, redundancy weighting w_{red}, sinogram-domain weighting w_{sino}, 2D backprojection A_{2d}^T, and detector-domain weighting w_d. Among these components, only w_{sino} is parameterized and learned during training.

The differentiable nature of this pipeline enables end-to-end optimization of w_{red} in a data-driven manner, making it adaptable to complex and irregular scan trajectories. However, since these weights are tightly coupled to the given trajectory, retraining is required whenever the scan trajectory changes. To address this, we introduce a trajectory standardization framework that enables the reuse of learned weights across a class of geometrically consistent trajectories.

B. Trajectory Standardization

To ensure consistency across trajectories, we first identify a unique starting view, then reorder the remaining projections to enforce geometric continuity.

1) Starting View Selection: The initial view is determined by identifying the projection that corresponds to the maximum z-coordinate. In the event of a tie, the projection with the highest x-coordinate is selected, followed by the y-coordinate. This approach ensures that each trajectory is initiated with a unique perspective.

2) Projection Reordering: Subsequent to the selection of the initial view, the remaining projections are then reorganised according to their spatial proximity to the initial view. For each projection, the Euclidean distance to the current view is computed, and the next projection that is closest in space is selected. This process is repeated iteratively, thereby ensuring that the projections are ordered in a continuous manner and maintaining the spatial coherence of the trajectory. The resulting ordered sequence of projections represents the standardized trajectory geometry, which is used for generating stimulated training data.

III. Experiments and Results

The evaluation of the proposed framework utilises the Random Orbit geometry settings from [4]. Specifically, the projection positions of the random trajectory are randomly reordered four times, thereby generating different experimental trajectory geometries. For these trajectories, the Trajectory Standardization method is applied to obtain a unique and consistent standard trajectory. Utilising these distinct trajectory geometries, synthetic datasets are generated for training, validation, and testing purposes, in accordance with the data generation procedure delineated in [4].

We train the differentiable SV-FBP model using the synthetic datasets derived from these trajectory geometries. To assess performance, we compare the reconstruction accuracy of the models trained on the four randomized trajectories with the model trained on the standardized trajectory. The evaluation is based on three metrics: Mean Squared Error (MSE), Structural Similarity Index (SSIM), and Peak Signal-to-Noise Ratio (PSNR).

Table I demonstrates that the model trained on the standardized trajectory yields results nearly identical to those trained on trajectories with randomly shuffled point orders. This finding suggests that using standardized trajectory geometry can significantly enhance the model's generalization capability without compromising accuracy. Furthermore, it demonstrates the negative impact on reconstruction quality when the point order is inconsistent with the assigned weights, further emphasising the importance of trajectory standardisation. In the table, each shuffled trajectory is identified by the specific random seed used for point-order permutation.

TABLE I
Comparison of Image Quality Metrics with Mean ± Standard Deviation

Trajectories	**MSE↓**	**PSNR (dB)↑**	**SSIM↑**
Shuffled(Seed0)	0.1201± 0.0193	34.72± 1.26	0.9178±0.0078
Shuffled(Seed1)	0.1222± 0.0198	34.58± 1.27	0.9125±0.0079
Shuffled(Seed2)	0.1219± 0.0195	34.59± 1.27	0.9139±0.0084
Shuffled(Seed3)	0.1200± 0.0195	34.74± 1.27	0.9169±0.0083
Standardized	0.1207± 0.0191	34.68± 1.28	0.9177±0.0081
Inconsistent	0.1446± 0.0207	33.09± 1.29	0.8666±0.0098

IV. Conclusion and Discussion

We proposed a trajectory standardization framework that enables a differentiable SV-FBP model to generalize across CBCT trajectories with consistent geometric structures. By deterministically selecting a starting view and reordering projections based on spatial proximity, such trajectories are transformed into a unified canonical form. The experimental results demonstrate that a model trained on this standardized trajectory performs comparably to those trained on individually randomized trajectories, without requiring retraining. This simple yet effective strategy greatly improves the practicality of SV-FBP models in settings with variable scan paths.

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